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Variations in eccentricity (e) and argument of perigee (ω) cause a significant effect on the ground-track repeatability of a satellite such as TOPEX/POSEIDON. In order to minimize this variation, e and ω should be maintained near their frozen values. The deviation from these frozen values corresponds to an off-set of the ground-track from its nominal path. Analytical expressions have been found to express this relationship while keeping an arbitrary point of the ground-track fixed. The initial off-set determines the subsequent evolution of e and ω about their frozen values. This long-term behavior was numerically determined using an Earth gravitational field including the first 17 zonal harmonics. The numerical results were plotted together with the analytical constraints to see if the later values of e and ω cause unacceptable deviation in the ground-track. Additional tests were done by also including the Earth tesseral terms, the long period luni-solar effects and solar radiation pressure in the force model.

PURPOSE and METHOD

The TOPEX/POSEIDON mission requires strict control of the ground-track. A reference ground track has been chosen which corresponds to an orbit which has been propagated with all forces which can be included and still maintain the 10-day repeat cycle (see Reference 1). The presence of other forces such as drag and luni-solar gravitational effects will necessitate maintenance maneuvers at varying intervals to maintain the orbit within $\pm$ 1 Km of this reference. It is hoped that these maneuvers will be small and adjust the semi-major axis only. However, as described below, there is a possibility that the orbit eccentricity (e) and/or argument of perigee (ω) might also have to be adjusted to minimize the effect of deviations of these elements from their nominal values.

For a given altitude and inclination there exists an e and ω such that these values will remain constant when the satellite is acted upon by certain perturbative forces (see Reference 2). Such a frozen orbit would be desired for TOPEX/POSEIDON to minimize the effect e and ω variations on the ground-track. However, in the actual mission the exact frozen values of mean e and ω will be neither achievable or maintainable because of errors in the orbit determination and maneuver execution and the presence of forces such as drag. Instead, mean values close to the frozen values of $e = 7.5 \times 10^{-5}$ and $\omega = 90°$ will be targeted to initially and perhaps later during maintenance maneuvers. The difference between the values obtained and the nominal values will cause an immediate deviation from the targeted ground track (TRGT). Because of the initial off-set, there is also a long-term variation of e and ω which could cause even greater ground track deviation.

The amount of ground track off-set at the ascending and descending nodes for changes to both e and ω was determined analytically. It should be kept in mind that the deviations at other points in the ground track could be somewhat larger and are of equal importance to the fundamental concept of the mission. Also note that a 100 m limit was used for acceptable ground-track deviation. This value is arbitrary but is representative of the portion of the current error budget for ground-track control assigned to non-drag effects.

After determining the loci of values for e and ω which produced the 100 m off-set, a numerical integration was performed to create a curve which depicts the behavior of e and ω over an extended (2 year) period of time. Initial conditions very close to the frozen values would have given the usual oval shape centered around the frozen point. However, the slightly further off-set initial conditions that were chosen (see below) resulted in the egg-shaped curves shown on the figures. This is due to the fact that the eccentricity cannot be less than zero. These initial conditions were chosen because they indicated the shape of the curve when it was skirting the 100m ground track off-set areas for the whole time period.

ANALYTICAL BOUNDARY VALUES

The analytical method of defining the e-ω limits was to determine the change in the ascending and descending nodal crossing times (Δt_A, Δt_D) as a function of Δe and $\Delta\omega$ when the time of passage at an arbitrary value of true anomaly (f) of the unperturbed orbit was kept constant. This results in:

$$\Delta t_A = \frac{2}{n} [e(1 + \sin f) - (e + \Delta e) (\cos \Delta\omega + \sin f \cos \Delta\omega - \cos f \sin \Delta\omega)]$$
$$\Delta t_D = \frac{2}{n} [e(1 - \sin f) - (e + \Delta e) (\cos \Delta\omega - \sin f \cos \Delta\omega + \cos f \sin \Delta\omega)]$$

where n is the mean motion.

Two cases were chosen to be investigated. Consider the case where f=0 which implies that the time of passage at the highest latitude (the original perigee) remains fixed. Thus both ascending and descending nodal crossings are affected by Δe and $\Delta\omega$. However, note that the cases of $f=\pm \pi/2$ where either the time of ascending or descending nodal crossings is fixed, actually in general, produce larger ground track deviations at the opposite node as compared to the f=0 case. For example, a Δe and a $\Delta\omega$ for the f=0 case might produce a 40m deviation east at the ascending node and 60m west at the descending node while the same perturbation with t_A fixed would produce a 100m deviation at the descending node.

Nevertheless, the f=0 case is a good representation of the expected behavior, since there is no reason why ground track control would be maintained at the the ascending node at the expense of the descending nodes or vice-versa. With f=0 the above equations become:

$$\Delta t_A = \frac{2}{n}\,[e-(e + \Delta e)\,(\cos \Delta\omega - \sin \Delta\omega\,)]$$
$$\Delta t_D = \frac{2}{n}\,[e-(e + \Delta e)\,(\cos \Delta\omega + \sin \Delta\omega\,)]$$

The value $\Delta t = 0.2155$ seconds corresponding to 100m change in the ground-track due to the Earth's rotation can be substituted into both equations. The current Operational Orbit for TOPEX/POSEIDON has $n = 9.314 \times 10^{-4}$ and $e = 9.5 \times 10^{-5}$ .These give the loci of points in e-ω space which represent the boundary of interest. This is evident on Figure 1, where the shaded area represents a ground track deviation of greater than 100m.

For the more restrictive case when $f=\pm\pi/2$, the time of passage of one node is held fixed while the change at the other node is given by:

$$\Delta t_x = \frac{4}{n}\,[e - (e + \Delta e)\cos \Delta\omega\,]$$

which when again a Δt corresponding to 100m offset is substituted, produces the boundary area given in Figure 2. Finally, as a future study, it is hoped that a general formulation can be found which gives the ground-track constraint on e and ω independent of the value of f.

NUMERICAL PROPAGATION RESULTS

The above analysis determined the theoretical boundaries for a 100m offset. They can be used with actual values of the e and ω over an extended period of time. Thus it can be determined whether an initial offset from the frozen values while causing an acceptable ground-track offset initially, does not cross the 100m boundary later in the subsequent evolution of the orbit. The first analysis used the first 17 zonal terms of the Earth's gravity field as the force model and propagated the average equations of motion. Thus the values of e and ω are strictly mean in the sense of containing no periodic terms.

Note that it is known that the curves of mean e-ω are well behaved in the sense that they form concentric shapes around the frozen values point so that any initial conditions inside a given curve will guarantee that the following values of e and ω will remain inside that curve. Therefore the initial conditions of $\Delta e = 0$ and $\Delta\omega = 42.71°$ were chosen which are on the 100m boundary for the f=0 case. The subsequent 2-year propagation is indicated by the egg-shaped curve on Figure 1. Note that as the later values of e and ω progressed around the centroid frozen values, the ground-track off-set remained close to the 100m limit.

Similarly, the initial conditions $\Delta e = 5.018 \times 10^{-5}$ and $\Delta\omega = 0$ were chosen on the 100m boundary for the $f=\pm\pi/2$ case. Again a 2-year propagation produced the curve shown in the e-ω space of Figure 2. Again the later values of e and ω remained close to the boundary.

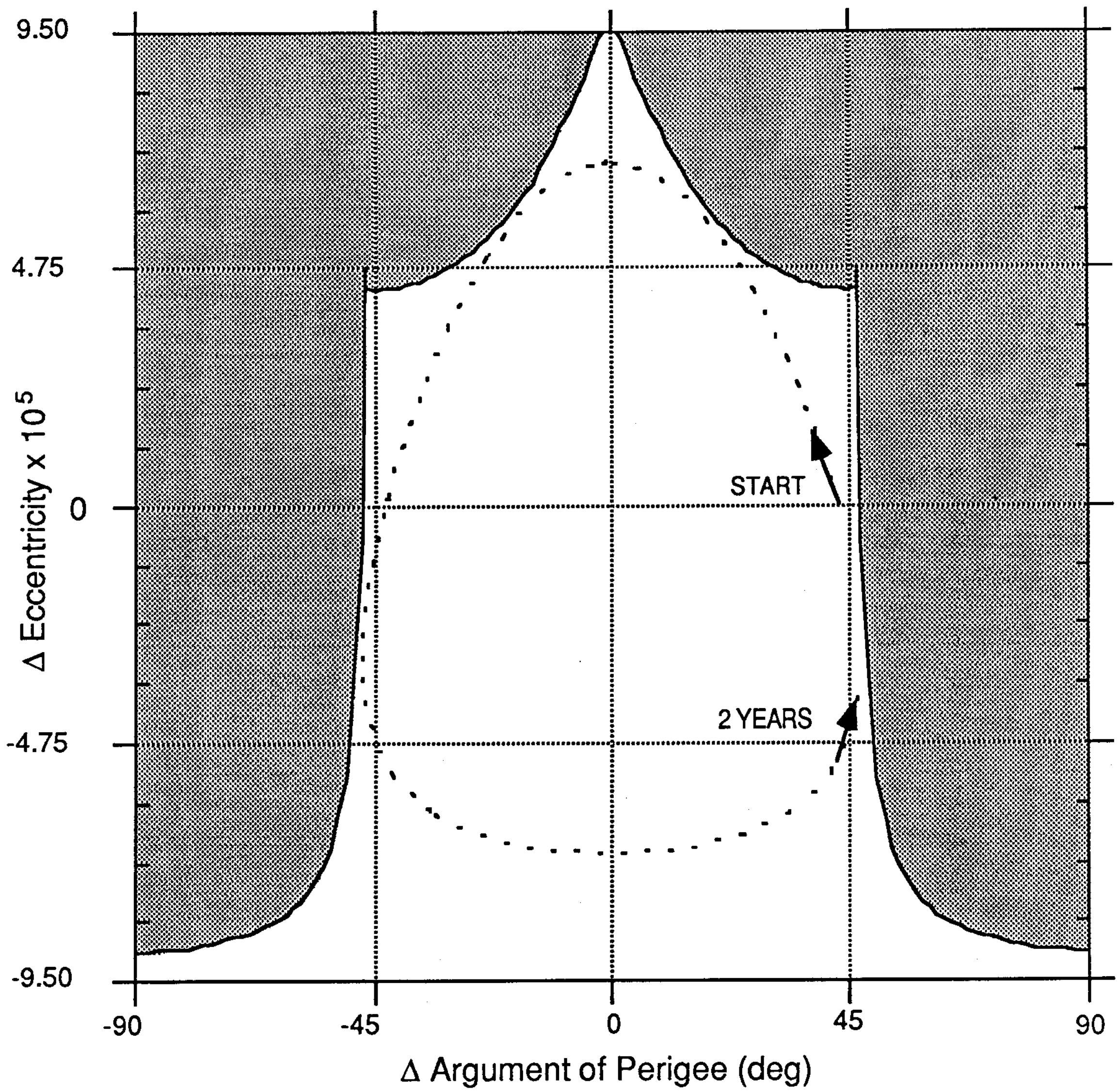

Figure 1. Constant Perigee Passage (f=0)

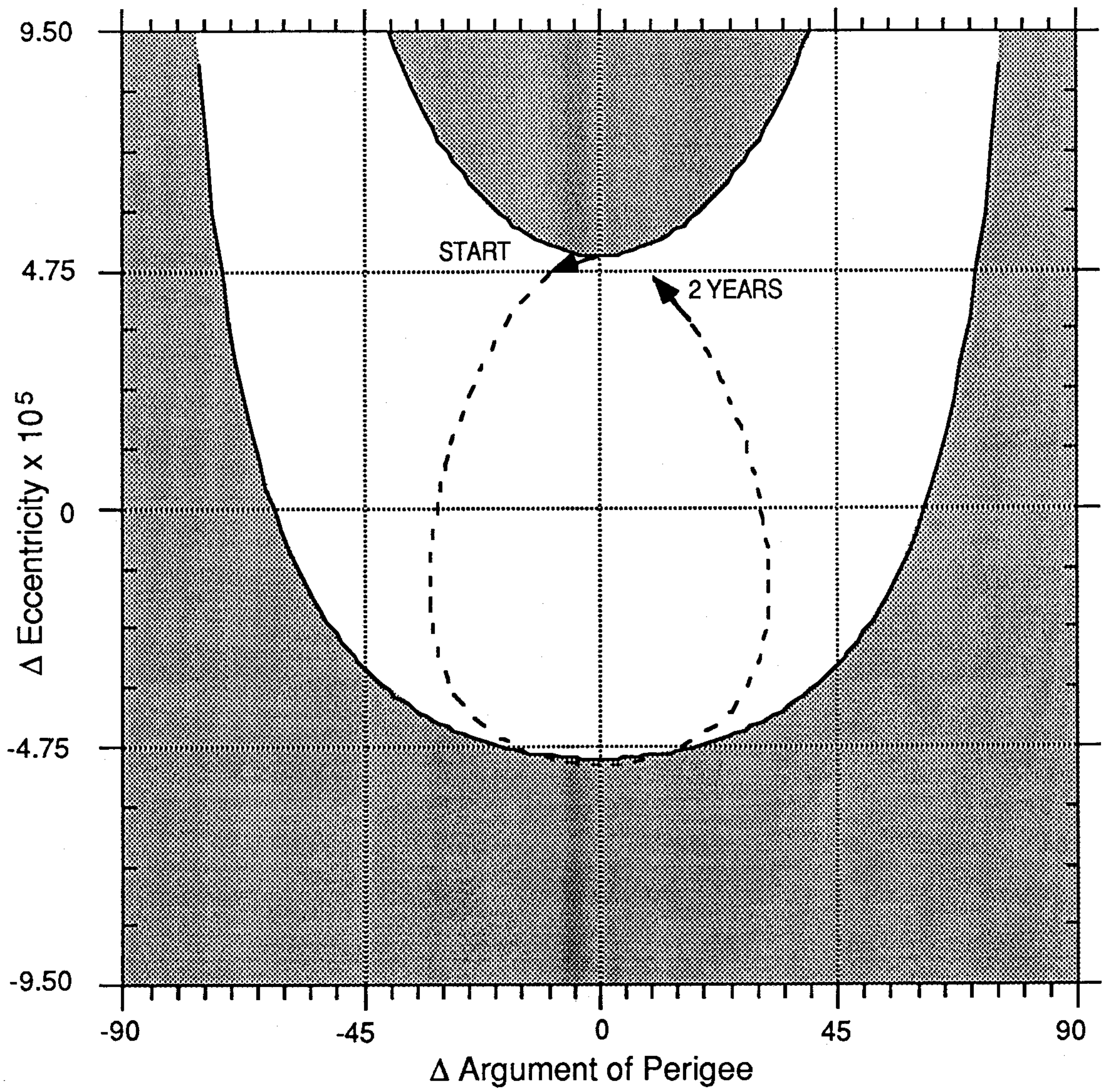

Figure 2. Constant Node Passage ($f=\pm\pi/2$)

ADDITIONAL PERTURBATION RESULTS

The encouraging results of the above propagation prompted the investigation to see if long term effects from various perturbations would cause the ground-track to deviate away from the frozen values to a significant amount. The periodic perturbations considered are namely: the medium and long period terms of the tesseral part of the Earth's gravity field, the long period terms caused by the luni-solar effects and the effect of solar radiation pressure (SRP).

The first step is to adjust the initial conditions to account for the periodic effects of interest at that epoch. Fortunately, there is a program (Reference 2) which can do this for both Earth tesseral terms and luni-solar effects. The orbit was then propagated for two years using the pertinent force field but still using the averaged equations of motion.

The results indicated that the addition of the first two types of perturbations had little effect on the behavior of e and ω. For both a 17 x 17 gravity field and the luni-solar perturbation the change in the ground-track was almost indistinguishable on the plots. However as Figure 3 indicates, the effect of the solar radiation pressure was significant. The center curve is the same as that of Figure 2. The other curve includes the long-term effect that the SRP has on both e and ω, as the plane of the orbit precesses with respect to the Sun. By transposing the values from the SRP curve to either Figure 1 or Figure 2, it is evident that the 100m boundary will be violated. Further investigation should be undertaken to see if the SRP effects on e and ω have to be considered when designing maintenance maneuvers.

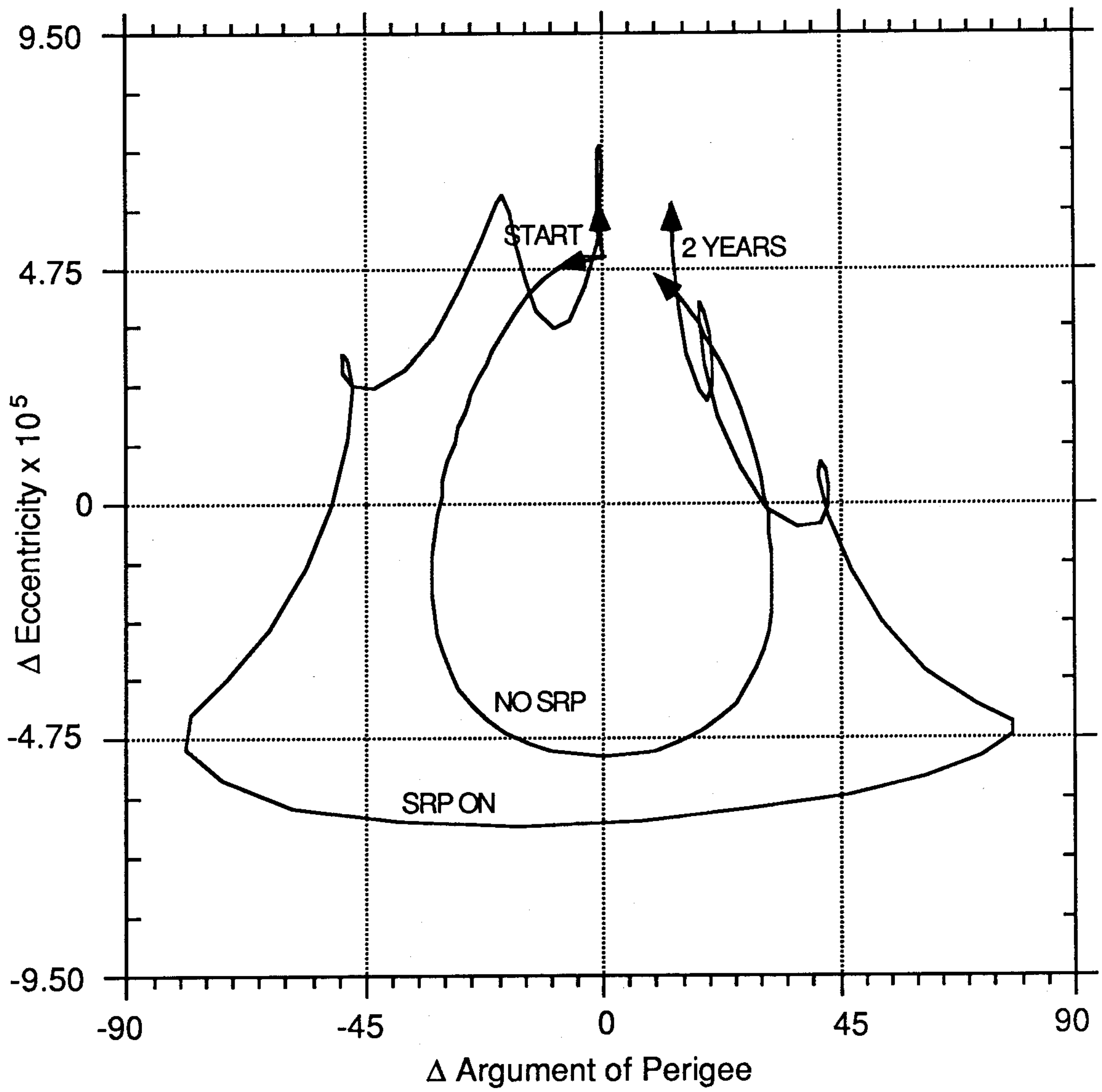

Figure 3. The Effect of Solar Radiation

SUMMARY

The two cases where the highest latitude crossing time and one of the equator crossings were held constant were investigated to gain understanding into the problem of e-ω control for TOPEX/POSEIDON. When considering Earth gravity only, the accompanying figures indicate that off-sets in e and ω that are inside the integrated curve are not only less than 100m initially but remain so throughout time. Obtaining values inside the curve would be a good target strategy. The importance of considering the combination of Δe and $\Delta\omega$ is evident however the values of Δe less than $5x10^{-5}$ and $\Delta\omega$ less than 35° can be used as figures of merit for design purposes. The ability to achieve this target will depend upon such things as orbit determination accuracy and errors in Osculating-to-Mean Element conversion and maneuver execution. The most interesting result is the significant effect that solar radiation pressure has on the mean values of e and ω. Two possible repercussions would be to have either more stringent targeting on the values of e and ω or to include the SRP effects in the design of maintenance maneuvers. This is a good topic for further investigation. The result of adding tesseral terms of the Earth gravity field or long period luni-solar effects on e and ω was negligible and thus will not have to be considered in maintaining the ground-track. Of course, there are significant effects such as the perturbation of the Sun and Moon on the inclination which do effect the ground-track significantly. These effects are the subject of other studies.

REFERENCES

1. The Inclusion of Higher Degree and Order Gravity Field Terms in the Design of a Repeat Ground Track," M. A. Vincent, AIAA/AAS Astrodynamics Conference Paper No. AIAA-90-2899-CP, Portland OR, August 20-22, 1990

2. "Analysis and Application of Frozen Orbits for the TOPEX Mission," J.C. Smith, AIAA/AAS Astrodynamics Conference Paper No. AIAA 86-2069-CP, Williamsburg, VA, August 18-20, 1986

3. "Periodic Gravitational Perturbations for Conversion Between Osculating.and Mean Orbital Elements," J. Guinn, AAS/AIAA Astrodynamics Conference Paper No. AAS 91-430, Durango, CO, August 19-22, 1991